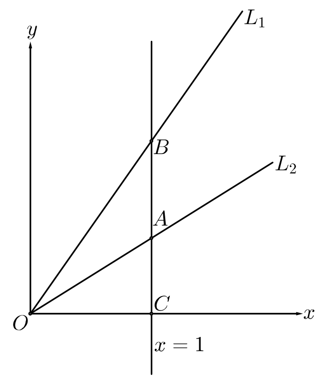
11. Answer: -110. The formula for the sum S_n of n terms of an arithmetic progression, whose first term is a and whose common difference is d, is $2S_n = n(2a+(n-1)d)$. Therefore,

$$\begin{aligned} 200 &= 10(2a+9d) \\ 20 &= 100(2a+99d) \\ 2S_{110} &= 110(2a+109d) \end{aligned}$$

Subtracting the first equation from the second and dividing by 90 yields $2a + 109d = -2$. Hence, $2S_{110} = 110(-2)$, so $S_{110} = -110$.

12. **Answer (C):**



In the adjoining figure, L_1 and L_2 intersect the line $x = 1$ at B and A, respectively; C is the intersection of the line $x = 1$ with the x-axis. Since $OC = 1$, AC is the slope of L_2 and BC is the slope of L_1. Therefore, $AC = n, BC = m$, and $AB = 3n$. Since OA is an angle bisector

$$\frac{OC}{OB} = \frac{AC}{AB}.$$

This yields $\frac{1}{OB} = \frac{n}{3n}$ and $OB = 3$.

By the Pythagorean theorem $1 + (4n)^2 = 9$, so $n = \frac{\sqrt{2}}{2}$. Since $m = 4n$, $mn = 4n^2 = 2$.

OR

Let θ_1 and θ_2 be the angles of inclination of lines L_1 and L_2, respectively. Then $m = \tan\theta_1$ and $n = \tan\theta_2$. Since $\theta_1 = 2\theta_2$ and $m = 4n, 4n = m = \tan\theta_1 = \tan 2\theta_2 = \frac{2\tan\theta_2}{1-\tan^2\theta_2} = \frac{2n}{1-n^2}$. Thus $4n(1-n^2) = 2n$. Since $n \neq 0$, $2n^2 = 1$, and mn, which equals $4n^2$, is 2.

13. **Answer (B):** If the bug travels indefinitely, the algebraic sum of the horizontal components of its moves approaches $\frac{4}{5}$, the limit of the geometric series

$$1 - \frac{1}{4} + \frac{1}{16} - \cdots = \frac{1}{1 - (-\frac{1}{4})}.$$

Similarly, the algebraic sum of the vertical components of its moves approaches $\frac{2}{5} = \frac{1}{2} - \frac{1}{8} + \frac{1}{32} \cdots$. Therefore, the bug will get arbitrarily close to $\left(\frac{4}{5}, \frac{2}{5}\right)$.

OR

The line segments may be regarded as a complex geometric sequence with $a_1 = 1$ and $r = \frac{i}{2}$. Thus

$$\sum_{i=1}^{\infty} a_i = \frac{a_1}{1-r} = \frac{2}{2-i} = \frac{4+2i}{5}.$$

In coordinate language, the limit is the point $\left(\frac{4}{5}, \frac{2}{5}\right)$.

14. Answer: -3. For all $x \neq -\frac{3}{2}$,

$$x = f(f(x)) = \frac{c\left(\frac{cx}{2x+3}\right)}{2\left(\frac{cx}{2x+3}\right) + 3} = \frac{c^2 x}{2cx + 6x + 9},$$

which implies $(2c+6)x + (9 - c^2) = 0$. Therefore, $2c + 6 = 0$ and $9 - c^2 = 0$. Thus, $c = -3$.

15. **Answer (B):** Let m be the price of the item in cents. Then $(1.04)m = 100n$. Thus $(8)(13)m = (100)^2 n$, so $m = (2)(5)^4 \frac{n}{13}$. Thus m is an integer if and only if 13 divides n.

16. **Answer (B):** The edges of the tetrahedron are face diagonals of the cube. Therefore, if s is the length of an edge of the cube, the area of each face of the tetrahedron is

$$\frac{(s\sqrt{2})^2\sqrt{3}}{4} = \frac{s^2\sqrt{3}}{2},$$

and the desired ratio is

$$\frac{6s^2}{4\left(\frac{s^2\sqrt{3}}{2}\right)} = \sqrt{3}.$$

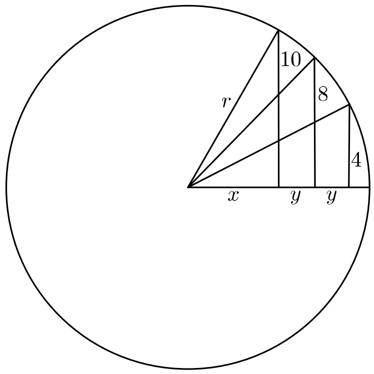
17. Answer: 3. Since $i^2 = -1$,

$$(n+i)^4 = n^4 - 6n^2 + 1 + i(4n^3 - 4n).$$

This is real if and only if $4n^3 - 4n = 0$. Since $4n(n^2 - 1) = 0$ if and only if $n = 0, 1, -1$, there are only three values of n for which $(n+i)^4$ is real; $(n+i)^4$ is an integer in all three cases.

18. **Answer (D):** $\log_b \sin x = a; \sin x = b^a;$
$\sin^2 x = b^{2a}; \cos x = (1 - b^{2a})^{\frac{1}{2}};$
$\log_b \cos x = \frac{1}{2} \log_b(1 - b^{2a})$.

19. **Answer (D):**



The adjoining figure is constructed from the given data. We let r be the radius, x the distance from the center of the circle to the closest chord, and y the common distance between the chords. The Pythagorean theorem provides three equations in r, x, and y:

$$r^2 = x^2 + 10^2$$
$$r^2 = (x + y)^2 + 8^2$$
$$r^2 = (x + 2y)^2 + 4^2.$$

Subtracting the first equation from the second yields $0 = 2xy + y^2 - 36$, and subtracting the second equation from the third yields $0 = 2xy + 3y^2 - 48$. Equating the right sides of these last two equations and collecting like terms yields $2y^2 = 12$. Thus, $y = \sqrt{6}$; and by repeated substitutions into the equations above, $r = \frac{5\sqrt{22}}{2}$.

20. **Answer (C):** The number of ways of choosing 6 coins from 12 is $\binom{12}{6} = 924$. "Having at least 50 cents" will occur if one of the following cases occurs:

(1) Six dimes are drawn.
(2) Five dimes and any other coin are drawn.
(3) Four dimes and two nickels are drawn.

The number of ways (1), (2), and (3) can occur are $\binom{6}{6}$, $\binom{6}{5}\binom{6}{1}$ and $\binom{6}{4}\binom{4}{2}$, respectively. The desired probability is, therefore,

$$\frac{\binom{6}{6} + \binom{6}{4}\binom{4}{2} + \binom{6}{5}\binom{6}{1}}{924} = \frac{127}{924}.$$